CS400 assignment 3
Intersection writeup: Quadric Surfaces

Marc ten Bosch

December 15, 2004

1 Equation and Scene Format

Quadric Surfaces are second-order algebraic surface given by the general equation

$$Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + Iz + J = 0$$

Which can be written in matrix form

$$\mathbf{x}^T \mathbf{Q} \mathbf{x} = 0$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \mathbf{x}^T = [x, y, z, 1] \qquad \mathbf{Q} = \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix}$$

Note that $\mathbf{x}^T\mathbf{Q}\mathbf{x}$ is the dot product of $\mathbf{x}$ and $\mathbf{Q}\mathbf{x}$. The objects are specified in the scene file as:

```
QUADRIC A B C D E F G H I J <surface properties>
```

2 Intersection calculation

Substituting the ray equation

$$\rho(t) = \mathbf{P_0} + t\mathbf{d_0}$$

Into the quadric's equation

$$\mathbf{x} \cdot \mathbf{Q}\mathbf{x} = 0$$

yeilds

$$(\mathbf{P_0} + t\mathbf{d_0}) \cdot \mathbf{Q}(\mathbf{P_0} + t\mathbf{d_0}) = 0$$

Distributing the matrix multiplication we get

$$(\mathbf{P_0} + t\mathbf{d_0}) \cdot (\mathbf{Q}\mathbf{P_0} + t\mathbf{Q}\mathbf{d_0}) = 0$$

By distributing the dot product we get

$$\mathbf{P_0} \cdot \mathbf{QP_0} + t\mathbf{d_0} \cdot \mathbf{QP_0} + t\mathbf{P_0} \cdot \mathbf{Qd_0} + t^2\mathbf{d_0} \cdot \mathbf{Qd_0} = 0$$

$$\mathbf{P_0} \cdot \mathbf{QP_0} + 2t\mathbf{d_0} \cdot \mathbf{QP_0} + t^2\mathbf{d_0} \cdot \mathbf{Qd_0} = 0$$

Rearranging this equation, we get the quadratic equation

$$at^2 + bt + c$$

where

$$a \equiv \mathbf{d_0}^T\mathbf{Qd_0}$$
$$b \equiv 2\mathbf{d_0}^T\mathbf{QP_0}$$
$$c \equiv \mathbf{P_0}^T\mathbf{QP_0}$$

Which can be solved the exact same way as the Ray-Sphere intersection.

3 Surface normal computation

Once the intersection point r is calculated, we take the gradient of the quadric implicit equation

$$f(x,y,z) : Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + Iz + J = 0$$

To get the normal at any point x,y,z

$$N = \begin{bmatrix} \frac{\partial}{\partial x} f(x,y,z) \\ \frac{\partial}{\partial y} f(x,y,z) \\ \frac{\partial}{\partial z} f(x,y,z) \end{bmatrix}$$

$$= \begin{bmatrix} 2(Ax + By + Cz + D) \\ 2(Bx + Ey + Fz + G) \\ 2(Cx + Fy + Hz + I) \end{bmatrix}$$

And plugin $r = [r_x, r_y, r_z]^T$. Note that N must be normalized and therefore, the multiplication by two can be deleted.